Stochastic differential Equations to model fishing vessels displacements

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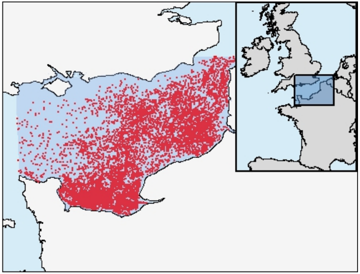
Fisheries Context

Modeling vessels displacement to

- Understand the spatial mechanisms of fishing effort;
- Getting indirect information over target species;
- Anticipate fishermen's reaction to management measure.

Case of study

GPS pings



Data Characteristics

- Mandatory Vessel Monitoring System (VMS);
- *Zone of interest* : Eastern Channel;
- All french bottom trawlers > 12 m;
- One ping every 1 or two hours (mostly irregular);
- 5 years of data.

Mathematical framework

Modeling objectives

- Dealing with irregularity in data;
- Dealing with a spatially explicit model;
- Having a general mathematical framework to express the movement.

Equations of the movement

- The position process $(X_t)_{t \geq 0}$ of a vessel is supposed to be the solution of a time homogeneous Stochastic Differential Equation (SDE)

$$dX_t = b(X_t)dt + \gamma(X_t)dW_t$$

  where W_t is the standard Wiener process.
- The drift $(b(X_t, t) : \mathbb{R}^2 \mapsto \mathbb{R}^2)$ and the diffusion $(\gamma(X_t, t) : \mathbb{R}^2 \mapsto \mathbb{R}^2)$ satisfy certain regularity conditions

Parametric form for $b(X_t)$ and $\gamma(X_t)$

The diffusion part

- In a first approach, the diffusion part is assumed to be constant

$$\gamma(X_t) = \Gamma$$

The drift part

- Following Brillinger (2009), the drift is assumed to be the gradient of a potential

$$b(X_t) = -\nabla P(X_t)$$

  where $P(X_t) : \mathbb{R} \mapsto \mathbb{R}^2$ is the **potential function**, and ∇ is the gradient operator;
- This writing leads to an intuitive interpretation of the drift, which is the direction towards attractive zones.

Parametric form for $b(X_t)$ (2)

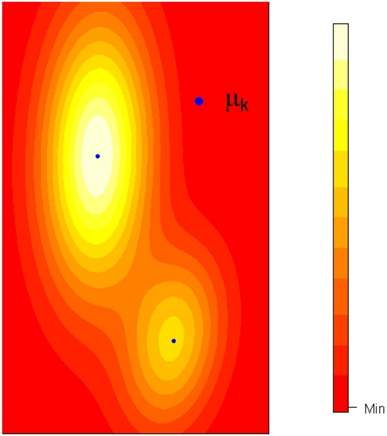
The potential function

- We pose $P(x) := \sum_{k=1}^{K} \pi_k \varphi_k(x)$
- φ_k being the probability density function of a 2 dimensional Gaussian random vector with mean μ_k and covariance matrix C_k
- π_k being the weight of the kth Gaussian component of the mixture.
- Thus, by definition of $b(x)$ we have

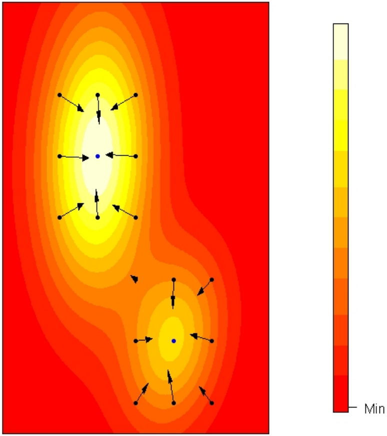
$$b(x) = \sum_{k=1}^{K} \pi_k \varphi_k(x) C_k^{-1} (x - \mu_k)$$

Parametric form for $b(X_t)$, example

The Potential function



The Drift function



Simulation of SDEs

How to draw a process from the p.d.f of our SDE solution?

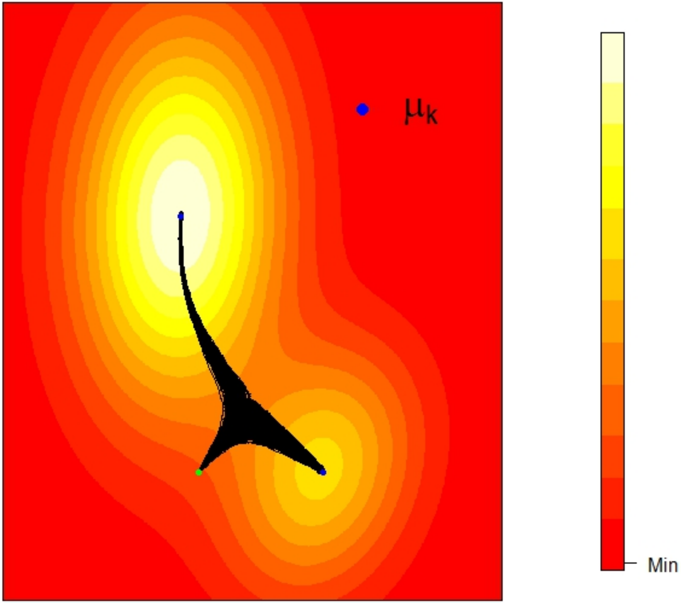
Two possible approaches

- **Discrete Schemes:** Samples are obtained from *approximation* (at regular discrete time steps) of the targeted p.d.f (e.g. Euler or Milstein scheme)
- **Exact Algorithm (EA):** Given some restrictions on $b(\cdot)$ and $\gamma(\cdot)$, samples are obtained from the *exact* targeted p.d.f.

Choice made for our model

- The $b(\cdot)$ and $\gamma(\cdot)$ described above satisfy the EA conditions.
- Simulations of processes are done using EA, avoiding an approximation of the targeted p.d.f.

Simulations Examples 1000 simulations



Estimation

Goal of the estimation

- Estimating Utilization Distribution parameters : $\Theta = \{\pi_k, \mu_k, C_k\}$
- Estimating diffusion parameter Γ
- The likelihood is then needed to be maximized.

Inference technique

- We can simulate from the targeted distribution;
- We don't have analytical expression of the likelihood
- A MCEM approach is adopted to maximize the likelihood.
- Work still in progress...

Conclusions and perspectives

Conclusions so far

- This general model seems promising to model movement;
- Allows to gather Utilization distribution concept and movement models.
- A challenging but not straightforward mathematical framework

Short term perspectives

- Application to our case of study
- Comparison with other methods to obtain UD.

Longer term perspectives

- Introducing environmental covariates in the drift function
- Introducing time heterogeneity

Some references

Brillinger, D. **Modelling Spatial Trajectories**, chp 26 *in* HANDBOOK OF SPATIAL STATISTICS, 2010, CRC Press.

Beskos, A., Papaspiliopoulos, O., Roberts, G.O. and Fearnhead, P. **Exact and computationally efficient likelihood-based estimation for discretely observed diffusion processes**, 2006, Journal of the Royal Statistical Society: Series B (Statistical Methodology).

Acknowledgments

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